Probabilistic Reasoning in Preschoolers: Random Sampling and Base Rate

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Abstract

Recent research in cognitive and language development suggests that infants and young children are capable of complex computations and statistical inference. The present studies investigated whether 4-year-old children can solve simple probabilistic reasoning problems. Two experiments investigated children's ability to use information from a sample to make generalizations about a population and vice versa. Results suggest that even young children can use the random sampling assumption and base rate information in simple probabilistic reasoning tasks. Future studies for addressing alternative interpretations and implications for learning and conceptual development are discussed.

Introduction

Research on cognitive and language development in recent decades has focused on specifying initial knowledge and concepts in infancy, and contemporary theories of development have emphasized the important role of innate constraints (e.g., Carey & Spelke, 1994; Spelke et al., 1992; Spelke, 1996). In the last few years, several laboratories have begun to investigate what kind of learning mechanisms are available to infants and young children, and how prior constraints and statistical information present in the input may be combined in rational ways (e.g., Aslin, Saffran, & Newport, 1998; Gerken, 2005; Gomez, 2002; Maye, Werker, & Gerken, 2002; Marcus, Vijiyan, Rao, & Vishton, 1999; Newport & Aslin, 2004; Saffran, Aslin, & Newport, 1996; Sobel, Tenenbaum, & Gopnik, 2004; Tenenbaum & Griffiths, 2001; Tenenbaum & Xu, 2000; Xu & Tenenbaum, 2005).

The current studies investigated whether young children are capable of rudimentary probabilistic reasoning. Previous studies in the Piagetian tradition of developmental psychology have tried to address some related issues. For example, Yost, Siegel, and Andrews (1962) asked whether 5-year-old children were able to use probability in making decisions, using a modified version of a task that Piaget had devised. Children were shown two boxes filled with marbles of different colors. One of the boxes had more red marbles than green ones, and the other had more green ones than red ones. The child was asked to choose a box if she wanted to draw a red marble. Children chose correctly at above chance level. Other studies have investigated whether older children were able to solve more complex probability tasks (e.g., Acredolo, O'Connor, Banks, and Horobin, 1989; Schlottman, 2001).

Our experiments focused on children who have not received any formal schooling. The question was whether they might already have certain intuitions about probabilities in the absence of any formal instructions. Experiment 1 asked whether 4-year-old children were able to use information from a sample to make generalizations about a population. Children were shown that a sample was drawn randomly from one of two boxes. They were then asked which box the sample came from given the ratio of the sample (e.g., the sample may have five yellow dog bones and one blue one. The two boxes had either mostly yellow dog bones or mostly blue dog bones). If the child assumes that the sample was drawn randomly from the box, then she would judge that it is more likely to get a sample of five yellow and one blue dog bones from the box that contained mostly yellow dog bones. Experiment 2 asked the children to use base rate information about a population to reason about a sample. Children were shown the content of a box (e.g., it contains mostly yellow dog bones). They were then asked which of two samples (e.g., one had five yellow ones and one blue one, and the other had one yellow one and five blue ones) came from the box. Again, if the child assumes that the sample was drawn randomly from the population, she would judge that the sample with five yellow ones and one blue one was more likely to have come from the box that contained mostly yellow dog bones.

Experiment 1

Method

Participants

Twelve preschoolers participated in this experiment (6 girls and 6 boys; mean age 4 years 0 months, ranging from 3 years 9 months to 4 years 4 months). Data from an additional four children were excluded from the final analyses due to either non-compliance with the procedure (1) or failure to complete the experiment (3). Participants were recruited from a database and their parents were contacted by phone and mail; they received no payment for participating. Their parents received either parking validation or transit reimbursement to cover travel expenses to the Baby Cognition Lab at the University of British Columbia from the Greater Vancouver Area. At the end of the experiment, the children received a small token gift such as a t-shirt, stuffed animal, or colouring book.

Materials

The materials used in this experiment included three pairs of boxes, three sets of objects, a large white foam core screen and a stuffed animal. The small stuffed animal was a pelican approximately six inches in height, and it played the role of a puppet that needed help playing a game. The screen measured 108 cm in width and 50 cm in height.

Stimuli. Gold and silver pinecones, red and green ball-shaped ornaments, and yellow and blue *Milkbone* dog bones served as the three sets of stimuli. The pinecones were approximately 13 cm in diameter, the ornaments were approximately 12.5 cm in diameter, and the dog bones were approximately 1 cm thick, 7 cm long and 2.5 cm wide.

Boxes. We created pairs of identical boxes, each filled with one set of objects. Each individual set of boxes and stimuli differed in color and size. All of the boxes were made from cardboard mailing boxes and the only differences between the sets of boxes were the colours and sizes of the materials used and the stimuli contained. The boxes that contained the pinecone stimuli were 20 x 31 x 41 cm. They were covered in green bristol board and the front side of each box was cut out leaving a 4 cm frame of mailing box and bristol board. We secured a transparent plastic covering to the inside of the frame to allow the child to see the contents of the box. This front window was then covered with purple foam boards attached to the bottom of the box with black tape and attached to the top with magnets so that the cover could be taken down during the experiment to allow the child to see the contents of the boxes. The ornament boxes were 31.5 cm cubes covered with blue bristle board and maroon front covers and the dog bone boxes were 22 x 26.5 x 31 cm covered with purple bristol board and maroon front covers.

We divided the inside of each box in half with a piece of black cardboard so that all boxes had two compartments. The front half of each box held the stimuli that served as the outcome of the trials for the child to view through the front window of the boxes. Within the pairs of boxes, the population stimuli differed only by the 5:1 ratio of one colour to the other. The splitting of the boxes into two compartments created the illusion that the boxes were completely full of stimuli when in reality, the back half of the boxes contained fixed samples to be pulled out during the experiment. This made it possible for the sample to appear to be drawn randomly. In the dog bone boxes, the mostly blue box contained 60 blue dog bones and 12 yellow ones, while the mostly yellow box contained 60 yellow dog bones and 12 blue ones. The pinecone and ornament boxes also contained 5:1 colour ratios of, 105:21 and 120:24 (number of items in each color), respectively.

An opening in the top of each box allowed the experimenter to reach into the back half of the boxes. This opening was covered with two pieces of opaque spandex material taped to the inside of the boxes so that they overlapped slightly in the middle. Containers with the desired samples to be pulled out during the trials sat in the back half of the boxes. For example, with the dog bone boxes, the box with mostly yellow dog bones in the front half had a sample of five yellow dog bones and one blue one in the back. The box with mostly blue dog bones in the front half had a sample of five blue dog bones and one yellow one in the back half. The boxes containing the pinecone and ornament stimuli had samples in the back set up in the same way.

Design

Each child completed 12 trials. On each trial, the experimenter pulled out a sample of six objects, five of one colour and one of the other colour from the back half of the boxes. The sampling appeared random to the child. The order of the different sets of objects was counterbalanced. At the beginning of each trial, two boxes were put on the table, one in front of the other. We randomized which box was placed in front and from which box the sample was removed. We also counterbalanced the sides that each box appeared on when the outcomes were revealed (see procedure below).

Procedure

Upon arrival, participants and their parents met with the experimenter in the lab waiting area where the child played with some toys and the parent filled out a consent form. Next, we informed the parent that the child would be playing with some boxes and a stuffed animal during the experiment. We asked the parents not to influence their child in any way during the study.

Once the parent and the child felt comfortable, they entered the experiment room where the child sat in a chair in front of a table facing the experimenter and the parent sat in a chair behind their child. In the corner of the room behind the child, a video camera recorded the session. The experimenter introduced "Pete" the pelican, "This is Pete the pelican, and Pete likes to play tricky games but he isn't very good at getting the right answer. Can you help Pete play a game today?" Next, familiarizations began: the experimenter showed the child each of the three pairs of boxes one at a time. The experimenter began with the dog bone boxes and put the two identical boxes onto the table. After this, she pulled down the front covers on each of the boxes and allowed the child to see that one box contained mostly blue dog bones and the other contained mostly yellow dog bones. Next, the experimenter asked the child, "What colours are the dog bones in the boxes?" and did not say anything else about the contents of the boxes (e.g. the ratio). This was done next for the ornament boxes and finally for the pinecone boxes in exactly the same manner.

After completion of the familiarization phase, the experimenter asked, "Are you ready to play the game with Pete now?" Once the child replied, test trials began. The experimenter picked up two identical boxes and put them on the table, one in front of the other. She shook one box back and forth a few times, closed her eyes, reached in, pulled out a handful of objects, and then repeated this action once more to reveal a six-piece sample, e.g. five blue dog bones and one yellow. Once she revealed the sample, the experimenter described it to the child by mentioning the numbers and colours of the stimuli ("Wow, there are five blue dog bones and one yellow one"). After this, the experimenter said, "Now I am going to hide the boxes and mix them around so that you and Pete can't tell which box those bones came from." The experimenter pulled up the large screen which was high enough that the child could not see over it and switched the boxes around for approximately seven seconds while saying "I'm switching the boxes around so that you and Pete can't tell which one the dog bones came from". She then removed the screen to reveal the boxes side by side and opened the covers to reveal their contents while saying, "Those dog bones (pointing at the sample) came from one of these two boxes. I can tell that Pete is stumped. Can you help him and point to which box you think they came from?" The child then pointed to the box that he/she believed the sample came from and the experimenter said, "Thank you". The experimenter placed the boxes under the table and the next trial began with another pair of boxes.

The experiment continued until all 12 trials were completed. On the last trial, the experimenter asked the child, "Why do you think all of those came from that box you pointed to?" This question aimed to see if children could articulate their reasoning and state explicitly why they had made the choices they did.

Each child took approximately fifteen minutes to complete all 12 trials and the final question. The experimenter then explained the experiment to the parent and allowed the child to choose a prize.

Data Analysis

The video recording of the experimental sessions was used for coding. On each trial, the child received one point for a correct response and zero points for an incorrect response. We also recorded verbatim each child's response to the final question from the videotape.

Results and Discussion

Children selected the correct box 74% of the time, which is reliably different from chance (50%), $t(11) = 4.4336$, $p < .005$. There were no differences across stimulus sets.

In order to succeed on this task, children may have made the assumption that the sample was randomly drawn from the population. With that assumption, the probability of drawing a sample of a certain ratio may be computed and an educated guess can be made. However, we do not know the precise nature of the computations given these results.

With the exception of one child, none of the children were able to articulate explicitly why they chose one box as opposed to the other, suggesting that the reasoning process was implicit and perhaps not accessible to verbal description or conscious reflection.

Experiment 2

Experiment 2 investigated whether children can reason about a sample when given information about the base rate of a population. The task is essentially the converse of that of Experiment 1. Instead of being asked which population the sample came from, the children were asked which one of two samples was drawn from a given population.

Method

Participants

Eighteen preschoolers participated in Experiment 2 (8 girls and 10 boys; mean age 4 years 1 month, ranging from 3 years 9 months to 4 years 5 months). Data from an additional five children were excluded from the final analyses due to inability to understand the meaning of the word "more" (3), non-compliance with the procedure (1), and parental interference (1). Recruitment was the same as in Experiment 1.

Materials

Experiment 2 used the same materials as Experiment 1 (Pete the puppet, the large screen, and the three pairs of boxes) with the addition of six small containers, some stickers and some cards. There were three pairs of identical containers, one pair for each pair of boxes, and they held the 6-piece samples during the trials. The containers for the ornaments and pinecones were made out of Plexiglas and measured 20 x 4.5 x 4 cm and 24.5 x 5 x 4 cm, respectively. The dog bone containers were made out of small cardboard boxes that were 18.5 x 8.5 x 5.5 cm with the top and front side removed. Each pair of containers held a sample including 1 piece of stimuli of 1 colour and 5 pieces of stimuli of the other colour (e.g. one dog bone container held 5 blue bones and 1 yellow and the other held 5 yellow bones and 1 blue). The stickers were small (none bigger than 3 cm^3) and assorted, and were given to the children as prizes throughout the game. The cards came from a deck of cards intended for a children's "Go Fish" game and were 12 x 9 cm with pictures of animals on them. They were given to the children at the beginning of the game for them to put their stickers on.

Design

Each child completed 12 test trials as in Experiment 1. The goal of Experiment 2 was to see if children can use base rate information to decide which of two samples was drawn from a population. We included a practice trial in Experiment 2, as the procedure was more difficult for the children to follow and we wanted to alleviate some of the mystery as to what was happening behind the large white screen. In the practice trial, we showed the child a box, asked them to describe its colours and stimuli, and asked which colour there were more of in the box to determine whether they understood the concept of "more or most". At the end of the practice trial, we also switched the placement of the two samples and asked them which ones came from the box again. This switch was done in order to demonstrate that the correct sample could appear on either side, preventing the child from assuming that the sample on the correct side during the practice trial must always be the correct sample.

The child then completed up to a maximum of 12 test trials. A test trial consisted of the experimenter pulling up one box from a pair onto the table and lowering the front cover of the box to allow the child to view its contents before closing the box. At the end of the trial, the child was required to point to one of two samples that they believed had come from the box. At the end of the 3rd, 6th, 9th, and 12th trials, the experimenter asked the child if she remembered what was in the box. This allowed us to detect whether the children were in fact able to remember what was in the box by the end of the trial. The child also chose a sticker as a prize on every third trial to provide them with some motivation for focusing on the game.

Across the 12 trials for each child, the order of stimuli presentation was counterbalanced in the same way as Experiment 1. In the first six trials, each child saw all six boxes and then saw them again in a different order in the last six trials. We also counterbalanced which side the correct sample appeared on. For the practice trial, we always used the box assigned to trial 3 for the counterbalance used for that particular child.

Procedure

The procedure was similar to Experiment 1 in that "Pete" the pelican was used in the same capacity and was introduced in the same way but with a slight addition to his character. The experimenter said, "This is Pete the Pelican. Pete loves to play tricky games, but he is not very good at getting the right answer. Pete needs you to help him play a fun game today because he isn't very good at remembering things. You need to have a good memory to do well at this game. Do you have a good memory?" This primed the children to be aware that they would need to pay close attention and use their memory. Familiarizations then proceeded exactly as in Experiment 1.

After the completion of the familiarization phase, the experimenter said, "Are you ready to play the game with Pete now?" Once the child responded, the experimenter said, "Good, now the first thing you get to do is choose one of these cards to put stickers on during the game. If you do a really good job of helping Pete you will get to choose some stickers every once in a while." The child then chose a card and placed it next to them, having the card with them throughout the game served as a reminder that they would be rewarded if they gave their best effort.

The experimenter then began with the practice trial. The experimenter pulled up the box, e.g. mostly silver pinecones, placed it on the table and opened the front cover. She then asked, "What do you see?" The children then typically answered by naming the objects and the colours. Then the experimenter asked, "Are there more silver pinecones or more gold ones in the box?" Once the child answered, the experimenter closed the box and presented one empty container and the sample of five gold pinecones and one silver pinecone in another container. She placed one container on either side of the box. She then pointed at the sample and said, "Those pinecones there came from Pete's house. Now lets shake up the box and take some out of there." She then pulled out five silver pinecones and one gold one from the top of the box and placed them in the empty container. This was done the same as in the test trials of Experiment 1, so that the sampling appeared to be randomly drawn. While pointing at the two samples, the experimenter said, "Now, Pete doesn't want to get these mixed up. This is an easy one, can you point to which group came from the box?" (Child points to a container.) "Thanks." Then, the experimenter switched the samples to the opposite sides. "And again, can you point to which ones came from the box?" (Child points.) "Thank you, and where did the other ones come from again?" Child usually responded with "Pete's house."

The experimenter then removed the box, said, "Okay, let's try one where you have to use your memory", and began the first test trial. For example, using the dog bones, the experimenter pulled up the mostly blue dog bone box and opened the front cover to allow the child to see the population. She then said, "Now look inside the boxes, can you tell me what is inside?" The child then responded and if they only said "dog bones", the experimenter asked what colours the dog bones were but did not mention anything else (e.g. the ratio). The experimenter then said, "Good, Pete needs you to remember exactly what the colours look like, take a picture with your brain so you really remember what is inside." The experimenter placed the large screen on the table saying, "I'm putting up this screen so that you and Pete can't see." She then shook the box around, said "I'm taking out some dog bones now", and picked up the two containers of pinecones from the floor where the child could not see and placed them on either side of the box. The experimenter then removed the screen and said to the child while pointing at the two samples, "Only one of these came from inside the box, can you help Pete and point to which one?" Once the child pointed, the trial was over and the next test trial began.

After the child pointed to a sample on the 3rd, 6th, 9th, and 12th trials, the experimenter asked, "Do you remember what was in the box?" The child typically responded by just naming the stimuli, so the experimenter then asked, "Were there more blue dog bones or yellow dog bones in the box?" Once the child replied the experimenter opened the box and allowed her to see if she were correct. If she was correct, the experimenter said, "Good job! Since you did such a good job of remembering, you win a sticker" and if the child were wrong, she said, "Oh no, well, that was a good try, you can pick a sticker anyway and we can try another one". After this, the experimenter said, "Okay, would you like to try a few more? And if you do a really good job, you'll have a chance to win some more stickers." If the child agreed, the experimenter completed three more trials. If the child did not want to continue, the experimenter said, "Okay. Well, you did a great job. You're all finished".

Each child took approximately ten to twenty minutes to complete the experiment. The experimenter then explained the study to the parent and allowed the child to choose a prize.

Data Analysis

Data analysis was the same as Experiment 1. A child received 1 point for a correct response and 0 points for an incorrect response. We also coded whether the child remembered the content of the box on the memory questions.

Results and Discussion

Not all children completed all 12 trials. Eight children completed all 12 trials, while seven children completed 9 trials and three children completed 6 trials. Because the memory demands were high in this task, we removed all trials where the children did not remember which colour there was more of in the box. Memory probes were administered on the 3rd, 6th, 9th, and 12th trials, therefore we included triplets of trials based on the child's answer on the memory probe. For example, if the child was incorrect on Trial 3 and correct on Trial 6 when answering the memory questions, we removed trials 1-3 and retained trials 4-6. Using this criterion, we eliminated 45 out of a total of 177 trials.

On average, children selected the correct sample 70% of the time, which is reliably different from chance (50%), $t(17) = 3.7985$, $p < .005$. There were no differences across stimulus sets.

These results suggest that children were able to keep track of the base rate information in the population. They were then able to compute the probability of drawing a sample of a certain ratio from the population. Again, we do not know the precise nature of the computations give these data.

General Discussion

Two experiments investigated whether 4-year-old children were able to solve simple probabilistic reasoning tasks. Experiment 1 showed that assuming random sampling, children were able to generalize from samples to populations. Experiment 2 showed that given base rate information about a population, children were able to predict which sample came from that population. Thus, even in the absence of formal instructions, children may already have certain intuitions that allow them to use probability in their reasoning.

These results corroborate the findings of Sobel, Tenenbaum, and Gopnik (2004). In their studies, 4-year-old children were given base rate information (in this case, whether there were a lot of objects that were 'blickets' in the population) and the results suggest that the children were able to take that information into account in a causal reasoning task.

Ongoing studies in our lab also provide preliminary evidence that even 8-month-old infants are able to engage in probability reasoning tasks. Using the violation-of-expectancy looking time method and a modified version of the tasks reported here, we found that infants looked longer at the less probable outcome (Xu, Garcia, & Kerlin, in preparation).

We have suggested here that children are sensitive to the sampling procedure as well as the base rate information in a population. However, we have not tested these assumptions directly. One possible alternative interpretation for these results is that instead of computing probabilities, children may have simply looked for a match between the sample and the population. This straightforward perceptual matching strategy only requires children to keep track of the global proportions (i.e., the ratio between the two colors in each box, or just which color there was more of) in order to succeed, without having to assume a random sampling procedure or to compute probabilities. Clearly, future studies are needed to address this alternative. One possibility may be to manipulate the sampling procedure. For example, we could compare a condition of random sampling (much like in Experiment 1) with a condition in which the experimenter looks into the box and draws out each item very carefully as if she had a particular kind of item in mind. Her deliberate choices may also be conveyed by saying phrases such as "Ah, I got what I wanted" or "Oops, wrong one" for each sampling. In the latter condition we predict that children would be at chance in choosing between the two populations. For the base rate experiment, we may convey to the child that the experimenter had a favorite color and she likes to have many items of that color. In this case we predict that children would always choose the sample with more items of the experimenter's favorite color.

In sum, our research makes two contributions to the literature on learning and conceptual development. First, it adds to the growing body of work cataloguing what learning mechanisms are available to infants and young children. To date we have evidence that infants and children are able to keep track of distributional information in the input (e.g., Maye et al., 2002), compute transitional probabilities (e.g., Aslin et al., 1998; Saffran et al., 1996), and extract rules from the input (e.g., Gerken, 2005; Marcus et al., 1999). Here we show that young learners may also able to use random sampling and base rate information in making probability judgments (see also Sobel et al., 2004). Second, the growing body of research on Bayesian models of cognition and language (e.g., Tenenbaum & Griffiths, 2001) depends on human learners possessing a certain level of competence in probabilistic reasoning. Our results demonstrate that even young children may have such competence. Furthermore, it seems likely that these basic reasoning abilities can be used in many domains of language and conceptual development. Future research will explore these issues in more detail.

Acknowledgments

We thank members of the Baby Cognition Laboratory at UBC for helpful discussion. We also thank the children and their parents for participating in the studies. This research was supported by an undergraduate summer research fellowship from the Natural Sciences and Engineering Research Council (NSERC) to S. Denison and a grant from NSERC to F. Xu.

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